NAME _____ DATE _____ PERIOD ____

10-4 Skills Practice

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. log 6 **0.7782**
2. log 15 **1.1761**
3. log 1.1 **0.0414**
4. log 0.3 **−0.5229**

Use the formula pH = −log[H+] to find the pH of each substance given its concentration of hydrogen ions.

5. gastric juices: $[H+] = 1.0 \times 10^{-1}$ mole per liter **1.0**
6. tomato juice: $[H+] = 7.94 \times 10^{-5}$ mole per liter **4.1**
7. blood: $[H+] = 3.98 \times 10^{-8}$ mole per liter **7.4**
8. toothpaste: $[H+] = 1.26 \times 10^{-10}$ mole per liter **9.9**

Solve each equation or inequality. Round to four decimal places.

9. $3^x > 243$ **$\{x|x > 5\}$**
10. $16^v \leq \frac{1}{4}$ **$\left\{v\middle|v \leq -\frac{1}{2}\right\}$**
11. $8^p = 50$ **1.8813**
12. $7^y = 15$ **1.3917**
13. $5^{3b} = 106$ **0.9659**
14. $4^{5k} = 37$ **0.5209**
15. $12^{7p} = 120$ **0.2752**
16. $9^{2m} = 27$ **0.75**
17. $3^{r-5} = 4.1$ **6.2843**
18. $8^{y+4} > 15$ **$\{y|y > -2.6977\}$**
19. $7.6^{d+3} = 57.2$ **−1.0048**
20. $0.5^{t-8} = 16.3$ **3.9732**
21. $42^{x^2} = 84$ **±1.0888**
22. $5^{x^2+1} = 10$ **±0.6563**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_3 7$ **$\frac{\log_{10} 7}{\log_{10} 3}$; 1.7712**
24. $\log_5 66$ **$\frac{\log_{10} 66}{\log_{10} 5}$; 2.6032**
25. $\log_2 35$ **$\frac{\log_{10} 35}{\log_{10} 2}$; 5.1293**
26. $\log_6 10$ **$\frac{\log_{10} 10}{\log_{10} 6}$; 1.2851**

Lesson 10-4

NAME _____ DATE _____ PERIOD ____

10-4 Practice (Average)

Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. log 101 **2.0043**
2. log 2.2 **0.3424**
3. log 0.05 **−1.3010**

Use the formula pH = −log[H+] to find the pH of each substance given its concentration of hydrogen ions.

4. milk: $[H+] = 2.51 \times 10^{-7}$ mole per liter **6.6**
5. acid rain: $[H+] = 2.51 \times 10^{-6}$ mole per liter **5.6**
6. black coffee: $[H+] = 1.0 \times 10^{-5}$ mole per liter **5.0**
7. milk of magnesia: $[H+] = 3.16 \times 10^{-11}$ mole per liter **10.5**

Solve each equation or inequality. Round to four decimal places.

8. $2^x < 25$ **$\{x|x < 4.6439\}$**
9. $5^a = 120$ **2.9746**
10. $6^z = 45.6$ **2.1319**
11. $9^m \geq 100$ **$\{m|m \geq 2.0959\}$**
12. $3.5^x = 47.9$ **3.0885**
13. $8.2^y = 64.5$ **1.9802**
14. $2^{b+1} \leq 7.31$ **$\{b|b \leq 1.8699\}$**
15. $4^{2x} = 27$ **1.1887**
16. $2^{a-4} = 82.1$ **10.3593**
17. $9^{z-2} > 38$ **$\{z|z > 3.6555\}$**
18. $5^{w+3} = 17$ **−1.2396**
19. $30^{x^2} = 50$ **±1.0725**
20. $5^{x^2-3} = 72$ **±2.3785**
21. $4^{2x} = 9^{x+1}$ **3.8188**
22. $2^{n+1} = 5^{2n-1}$ **0.9117**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_5 12$ **$\frac{\log_{10} 12}{\log_{10} 5}$; 1.5440**
24. $\log_8 32$ **$\frac{\log_{10} 32}{\log_{10} 8}$; 1.6667**
25. $\log_{11} 9$ **$\frac{\log_{10} 9}{\log_{10} 11}$; 0.9163**
26. $\log_2 18$ **$\frac{\log_{10} 18}{\log_{10} 2}$; 4.1699**
27. $\log_9 6$ **$\frac{\log_{10} 6}{\log_{10} 9}$; 0.8155**
28. $\log_7 \sqrt{8}$ **$\frac{\log_{10} 8}{2 \log_{10} 7}$; 0.5343**

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions $[H+]$ in the soil is not less than 1.58×10^{-8} mole per liter. What is the pH of the soil in which these irises will flourish? **7.8 or less**
30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk? **about 5000**
31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria? **about 5.6 h**
32. **SOUND** An equation for loudness L in decibels is given by $L = 10 \log R$, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise? **1000**

NAME _____ DATE _____ PERIOD ____

10-5 Skills Practice

Base e and Natural Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. e^3 20.0855
2. e^{-2} 0.1353
3. ln 2 0.6931
4. ln 0.09 −2.4079

Write an equivalent exponential or logarithmic equation.

5. $e^x = 3$ $x = \ln 3$
6. $e^4 = 8x$ $4 = \ln 8x$
7. $\ln 15 = x$ $e^x = 15$
8. $\ln x \approx 0.6931$ $x \approx e^{0.6931}$

Evaluate each expression.

9. $e^{\ln 3}$ 3
10. $e^{\ln 2x}$ $2x$
11. $\ln e^{-2.5}$ −2.5
12. $\ln e^y$ y

Solve each equation or inequality.

13. $e^x \geq 5$ $\{x|x \geq 1.6094\}$
14. $e^x < 3.2$ $\{x|x < 1.1632\}$
15. $2e^x - 1 = 11$ 1.7918
16. $5e^x + 3 = 18$ 1.0986
17. $e^{3x} = 30$ 1.1337
18. $e^{-4x} > 10$ $\{x|x < -0.5756\}$
19. $e^{5x} + 4 > 34$ $\{x|x > 0.6802\}$
20. $1 - 2e^{2x} = -19$ 1.1513
21. $\ln 3x = 2$ 2.4630
22. $\ln 8x = 3$ 2.5107
23. $\ln (x - 2) = 2$ 9.3891
24. $\ln (x + 3) = 1$ −0.2817
25. $\ln (x + 3) = 4$ 51.5982
26. $\ln x + \ln 2x = 2$ 1.9221

NAME _____ DATE _____ PERIOD ____

10-5 Practice (Average)

Base e and Natural Logarithms

Use a calculator to evaluate each expression to four decimal places.

1. $e^{1.5}$ 4.4817
2. ln 8 2.0794
3. ln 3.2 1.1632
4. $e^{-0.6}$ 0.5488
5. $e^{4.2}$ 66.6863
6. ln 1 0
7. $e^{-2.5}$ 0.0821
8. ln 0.037 −3.2968

Write an equivalent exponential or logarithmic equation.

9. $\ln 50 = x$ $e^x = 50$
10. $\ln 36 = 2x$ $e^{2x} = 36$
11. $\ln 6 \approx 1.7918$ $e^{1.7918} \approx 6$
12. $\ln 9.3 \approx 2.2300$ $e^{2.2300} \approx 9.3$
13. $e^x = 8$ $x = \ln 8$
14. $e^5 = 10x$ $5 = \ln 10x$
15. $e^{-x} = 4$ $x = -\ln 4$
16. $e^2 = x + 1$ $2 = \ln (x + 1)$

Evaluate each expression.

17. $e^{\ln 12}$ 12
18. $e^{\ln 3x}$ $3x$
19. $\ln e^{-1}$ −1
20. $\ln e^{-2y}$ $-2y$

Solve each equation or inequality.

21. $e^x < 9$ $\{x|x < 2.1972\}$
22. $e^{-x} = 31$ −3.4340
23. $e^x = 1.1$ 0.0953
24. $e^x = 5.8$ 1.7579
25. $2e^x - 3 = 1$ 0.6931
26. $5e^x + 1 \geq 7$ $\{x|x \geq 0.1823\}$
27. $4 + e^x = 19$ 2.7081
28. $-3e^x + 10 < 8$ $\{x|x > -0.4055\}$
29. $e^{3x} = 8$ 0.6931
30. $e^{-4x} = 5$ −0.4024
31. $e^{0.5x} = 6$ 3.5835
32. $2e^{5x} = 24$ 0.4970
33. $e^{2x} + 1 = 55$ 1.9945
34. $e^{3x} - 5 = 32$ 1.2036
35. $9 + e^{2x} = 10$ 0
36. $e^{-3x} + 7 \geq 15$ $\{x|x \leq -0.6931\}$
37. $\ln 4x = 3$ 5.0214
38. $\ln (-2x) = 7$ −548.3166
39. $\ln 2.5x = 10$ 8810.5863
40. $\ln (x - 6) = 1$ 8.7183
41. $\ln (x + 2) = 3$ 18.0855
42. $\ln (x + 3) = 5$ 145.4132
43. $\ln 3x + \ln 2x = 9$ 36.7493
44. $\ln 5x + \ln x = 7$ 14.8097

INVESTING For Exercises 45 and 46, use the formula for continuously compounded interest, $A = Pe^{rt}$, where P is the principal, r is the annual interest rate, and t is the time in years.

45. If Sarita deposits $1000 in an account paying 3.4% annual interest compounded continuously, what is the balance in the account after 5 years? $1185.30

46. How long will it take the balance in Sarita's account to reach $2000? about 20.4 yr

47. **RADIOACTIVE DECAY** The amount of a radioactive substance y that remains after t years is given by the equation $y = ae^{kt}$, where a is the initial amount present and k is the decay constant for the radioactive substance. If $a = 100$, $y = 50$, and $k = -0.035$, find t. about 19.8 yr

10-6 Study Guide and Intervention

Exponential Growth and Decay

Exponential Decay Depreciation of value and radioactive decay are examples of **exponential decay**. When a quantity decreases by a fixed percent each time period, the amount of the quantity after t time periods is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent decrease expressed as a decimal.

Another exponential decay model often used by scientists is $y = ae^{-kt}$, where k is a constant.

Example **CONSUMER PRICES** **As technology advances, the price of many technological devices such as scientific calculators and camcorders goes down. One brand of hand-held organizer sells for \$89.**

a. If its price decreases by 6% per year, how much will it cost after 5 years?

Use the exponential decay model with initial amount \$89, percent decrease 0.06, and time 5 years.

$y = a(1 - r)^t$ Exponential decay formula

$y = 89(1 - 0.06)^5$ $a = 89$, $r = 0.06$, $t = 5$

$y = \$65.32$

After 5 years the price will be \$65.32.

b. After how many years will its price be \$50?

To find when the price will be \$50, again use the exponential decay formula and solve for t.

$y = a(1 - r)^t$ Exponential decay formula

$50 = 89(1 - 0.06)^t$ $y = 50$, $a = 89$, $r = 0.06$

$\frac{50}{89} = (0.94)^t$ Divide each side by 89.

$\log\left(\frac{50}{89}\right) = \log (0.94)^t$ Property of Equality for Logarithms

$\log\left(\frac{50}{89}\right) = t \log 0.94$ Power Property

$t = \frac{\log\left(\frac{50}{89}\right)}{\log 0.94}$ Divide each side by log 0.94.

$t \approx 9.3$

The price will be \$50 after about 9.3 years.

Exercises

1. **BUSINESS** A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a \$500 item drop below \$100? **6 weeks**

CARBON DATING **Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of Carbon-14, t is the number of years ago the animal lived, and y is the remaining amount after t years.**

2. How old is a fossil remain that has lost 95% of its Carbon-14? **about 25,000 years old**

3. How old is a skeleton that has 95% of its Carbon-14 remaining? **about 427.5 years old**

10-6 Study Guide and Intervention *(continued)*

Exponential Growth and Decay

Exponential Growth Population increase and growth of bacteria colonies are examples of **exponential growth**. When a quantity increases by a fixed percent each time period, the amount of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is $y = ae^{kt}$, where k is a constant.

Example **A computer engineer is hired for a salary of \$28,000. If she gets a 5% raise each year, after how many years will she be making \$50,000 or more?**

Use the exponential growth model with $a = 28{,}000$, $y = 50{,}000$, and $r = 0.05$ and solve for t.

$y = a(1 + r)^t$ Exponential growth formula

$50{,}000 = 28{,}000(1 + 0.05)^t$ $y = 50{,}000$, $a = 28{,}000$, $r = 0.05$

$\frac{50}{28} = (1.05)^t$ Divide each side by 28,000.

$\log\left(\frac{50}{28}\right) = \log (1.05)^t$ Property of Equality of Logarithms

$\log\left(\frac{50}{28}\right) = t \log 1.05$ Power Property

$t = \frac{\log\left(\frac{50}{28}\right)}{\log 1.05}$ Divide each side by log 1.05.

$t \approx 11.9$ years Use a calculator.

If raises are given annually, she will be making over \$50,000 in 12 years.

Exercises

1. **BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find k for the growth formula $y = ae^{kt}$, where t is in minutes. **about 0.0175**

2. **INVESTMENT** Carl plans to invest \$500 at 8.25% interest, compounded continuously. How long will it take for his money to triple? **about 14 years**

3. **SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full? **about 5 years**

4. **EXERCISE** Hugo begins a walking program by walking $\frac{1}{2}$ mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day? **24 weeks**

5. **VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase? **about 14%**

Answers

NAME _____ DATE _____ PERIOD ____

10-6 Skills Practice

Exponential Growth and Decay

Solve each problem.

1. **FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline? **about 8.3 yr**

2. **INVESTING** Alex invests \$2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth \$3600? **10 yr**

3. **POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula $P = ae^{rt}$, find the expected population of the city in 30 years if the growth rate r of the population is 1.5% per year, a represents the current population in millions, and t represents the time in years. **about 5.5 million**

4. **POPULATION** The population P in thousands of a city can be modeled by the equation $P = 80e^{0.015t}$, where t is the time in years. In how many years will the population of the city be 120,000? **about 27 yr**

5. **BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%? **about 4.9 days**

6. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of k for this element? **about 0.00002845**

7. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS depreciated by a fixed rate of 6% and is now worth \$8600. How long ago did Abu buy the GPS system? **about 6.0 yr**

8. **BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$? **about 12.13%**

NAME _____ DATE _____ PERIOD ____

10-6 Practice (Average)

Exponential Growth and Decay

Solve each problem.

1. **INVESTING** The formula $A = P\left(1 + \frac{r}{2}\right)^{2t}$ gives the value of an investment after t years in an account that earns an annual interest rate r compounded twice a year. Suppose \$500 is invested at 6% annual interest compounded twice a year. In how many years will the investment be worth \$1000? **about 11.7 yr**

2. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000 if the growth rate per hour is 85%? **about 7.5 h**

3. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant k in the decay formula for the substance. **about 0.02166**

4. **DEPRECIATION** A piece of machinery valued at \$250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to \$100,000? **about 7.2 yr**

5. **INFLATION** For Dave to buy a new car comparably equipped to the one he bought 8 years ago would cost \$12,500. Since Dave bought the car, the inflation rate for cars like his has been at an average annual rate of 5.1%. If Dave originally paid \$8400 for the car, how long ago did he buy it? **about 8 yr**

6. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt-60, is radioactive and has a half-life of 5.7 years. Cobalt-60 is used to trace the path of nonradioactive substances in a system. What is the value of k for Cobalt-60? **about 0.1216**

7. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use $k = 0.00012$. **about 50,000 yr**

8. **POPULATION** The population of rabbits in an area is modeled by the growth equation $P(t) = 8e^{0.26t}$, where P is in thousands and t is in years. How long will it take for the population to reach 25,000? **about 4.4 yr**

9. **DEPRECIATION** A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally \$12,000, in how many months is it worth \$7350? **about 12 mo**

10. **BIOLOGY** In a laboratory, a culture increases from 30 to 195 organisms in 5 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$? **about 45.4%**